

# Bottom-tau unification by neutrinos in a nonsupersymmetric SU(5) model

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## Abstract

We show that Yukawa couplings of bottom quarks and tau leptons can be unified in a non-supersymmetric SU(5) model. We introduce an arbitrary number of right-handed neutrinos. Their masses and Yukawa couplings that satisfy the unification condition by renormalization group evolution are shown. In the case that the grand unification scale is  $10^{15.5}\text{GeV}$  and three right-handed neutrinos have the same mass, the upper bound on their mass is  $\sim 10^{14.1}\text{GeV}$ .

*Introduction.* Neutrino oscillations, which mean that neutrinos have masses, are evidence for physics beyond the Standard Model (SM). We know that at least two flavors of neutrinos have  $O(0.01\text{--}0.1)\text{eV}$  masses. Their Majorana masses can be written as  $v^2/M$  ( $v \simeq 174\text{GeV}$ ). We need  $M$  to be in the desert between the electroweak and Planck scales,  $O(10^{15})\text{ GeV}$  (seesaw scale). The neutrino masses can be induced by singlet, right-handed neutrinos (type-I [1–3]), an SU(2) triplet scalar (type-II [4–6]) or SU(2) triplet fermions (type-III seesaw [7]).

Near the seesaw scale, there may be another interesting phenomenon: unification of elementary forces [8, 9]. If the SM gauge group  $G_{\text{SM}} \equiv \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$  is embedded into one non-Abelian group, we can explain the charge quantization of quarks and leptons. The simplest grand unification theory (GUT) is the model based on the SU(5) gauge group [9], which is broken down to  $G_{\text{SM}}$  in one step. By solving renormalization group equations (RGEs), the three gauge couplings indeed evolve toward unification, but do not exactly meet at one scale. If some fields charged under  $G_{\text{SM}}$  exist between the electroweak scale and the GUT scale ( $M_G$ ), unification is still possible. The cases of a scalar representation  $15_H$  [10, 11] and a fermion representation  $24_F$  [12–15] are especially interesting since they can also explain the neutrino masses by type-II or type-I+III seesaw mechanisms and even the baryon asymmetry of the Universe.

In the SU(5) model, not only gauge couplings, but also eigenvalues of Yukawa coupling matrices of down-type quarks and charged leptons are unified at  $M_G$ . This unification condition cannot be satisfied if we consider known fields only [16]. In previous studies, a dimension five operator is assumed for this problem [13, 14, 17, 18]. The contribution of the operator is  $\lesssim vM_G/\Lambda$ .  $M_G$  is bounded from below by the nucleon decay search [19], naively  $M_G \gtrsim 10^{15.4}\text{GeV}$  [15]. The GUT scale near this bound is phenomenologically interesting and natural, because it may be tested by the nucleon decay search [20, 21] and is close to the seesaw scale. If the GUT scale is  $10^{15.5\text{--}16.0}\text{GeV}$ , and  $\Lambda \sim 10^{19}\text{GeV}$ , the higher-dimensional term gives  $\lesssim 0.1\text{GeV}$ . This is suitable for adjusting the first- and the second-generation Yukawa coupling unifications (and may explain why the  $u$  quark is lighter than the  $d$  quark), but is too small for that of the third generation. We need some mechanism for  $y_b = y_\tau$ , i.e.,  $b - \tau$  unification.

The main purpose of this letter is to show that  $b - \tau$  unification is possible if we introduce right-handed neutrinos to a non-supersymmetric SU(5) model. If there are right-handed neutrinos, the Yukawa couplings of neutrinos  $y_\nu$  changes the RGEs of Yukawa couplings of quarks and charged leptons [22, 23].  $b - \tau$  unification is not possible in the of type-II and type-III seesaw cases because an SU(2) triplet scalar and triplet fermions contribute to the running of  $y_\tau$  in the positive direction [18, 24]. We consider 1-loop beta functions throughout this letter mainly for a large experimental error of  $m_b$ . The gauge couplings are assumed to be unified by adjoint fermions and a scalar, but the details of gauge coupling unification do not essentially change our analysis of  $b - \tau$  unification.

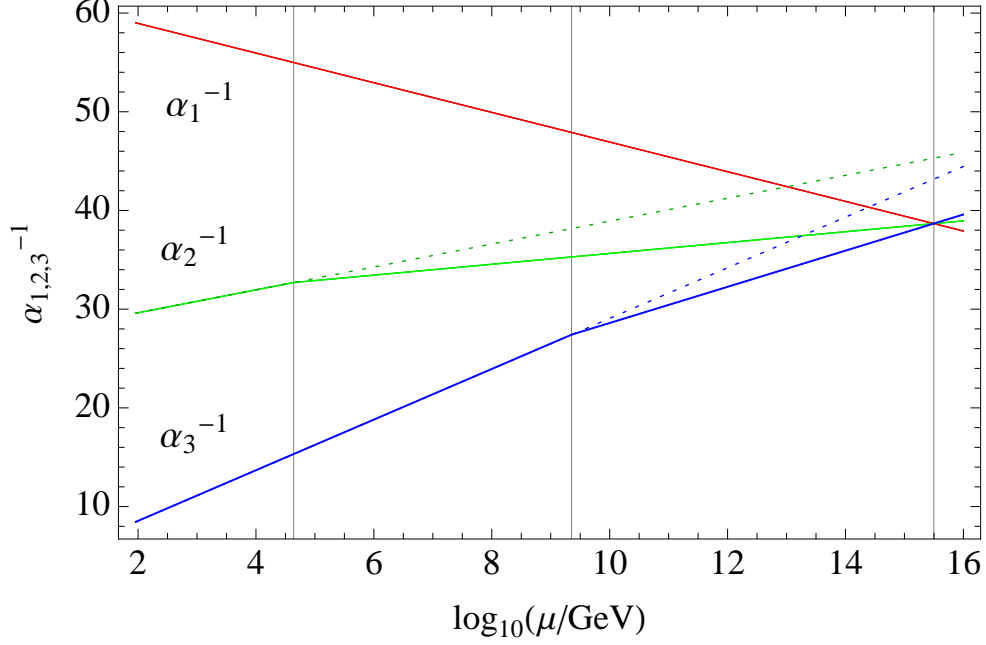


Figure 1: Gauge coupling running considering one standard deviation. The GUT scale is taken to be  $10^{15.5}\text{GeV}$ . Dotted lines show the SM case ( $\alpha_1$  does not change).

*Gauge and Yukawa coupling unifications.* For gauge coupling unification, we assume that three multiplets  $T_F$ ,  $T_H \equiv (1, 3, 0)$ ,  $O_F \equiv (8, 1, 0)$  (indicating representations under  $G_{\text{SM}}$  with  $F(H)$  meaning fermionic (Higgs) field) are much lighter than  $M_G$ . These fields can be embedded into adjoint representations of  $\text{SU}(5)$ , <sup>1</sup>  $24_{F,H} = (1, 1, 0) + (1, 3, 0) + (8, 1, 0) + (3, 2, -5/6) + (\bar{3}, 2, 5/6)$ . The singlet scalar in  $24_H$  breaks  $\text{SU}(5)$  down to  $G_{\text{SM}}$ . By the condition of gauge unification at  $M_G$ , we obtain

$$(m_{T_F}^4 m_{T_H})^{1/5} = M_Z \exp \left[ \frac{2\pi}{\Delta b_2} (\alpha_{1,Z}^{-1} - \alpha_{2,Z}^{-1}) + \frac{b'_2 - b_1}{\Delta b_2} t_G \right] = 4.37 \times 10^4 \text{GeV} \left( \frac{10^{15.5} \text{GeV}}{M_G} \right)^{\frac{84}{25}}, \quad (1)$$

$$m_{O_F} = M_Z \exp \left[ \frac{2\pi}{\Delta b_3} (\alpha_{1,Z}^{-1} - \alpha_{3,Z}^{-1}) + \frac{b'_3 - b_1}{\Delta b_3} t_G \right] = 2.25 \times 10^9 \text{GeV} \left( \frac{10^{15.5} \text{GeV}}{M_G} \right)^{\frac{91}{20}}. \quad (2)$$

Here we have used the experimental values  $\alpha = (127.940 \pm 0.014)^{-1}$ ,  $\alpha_3 = 0.1185 \pm 0.0006$ ,  $\sin^2 \theta_W = 0.23126 \pm 0.00005$  at  $\mu = M_Z = 91.1876 \text{GeV}$  [25] ( $\mu$  denotes the scale of renormalization). We have defined  $(b_1, b'_2, b'_3) \equiv (41/10, -3/2, -5)$ ,  $(\Delta b_2, \Delta b_3) \equiv (5/3, 2)$ ,  $t_G \equiv \ln(M_G/M_Z)$ . The running of gauge couplings in the  $M_G = 10^{15.5} \text{GeV}$  case is shown in Fig. 1. The light triplets can be searched for in collider experiments [13, 14, 26, 27]. By a condition  $m_{T_F}, m_{T_H} > M_Z$ , we get  $M_G < 10^{16.3} \text{GeV}$ . Note that this bound may be changed by higher loop corrections [28].

Next, we consider the running of Yukawa couplings. We assume that  $N_g$  singlet  $((1, 1, 0))$  fermions are coupled to left-handed neutrinos by Yukawa couplings  $Y_{\nu I i}$  ( $I = 1, \dots, N_g, i = 1, 2, 3$ ). We call these singlets right-handed neutrinos. They can be embedded into  $24_F$  or other multiplets. For simplicity, we assume that  $Y_{\nu I 3}$  are the dominant components and their absolute values are the same,  $|Y_{\nu I 3}| = y_\nu$ . We also consider the case in which all the masses of the right-handed neutrinos are the same  $M_N$ . The RGEs

<sup>1</sup>We need at least two generations of the adjoint fermion  $24_F$ . If  $T_F$  and  $O_F$  are components of the same generation of  $24_F$ , the other components such as  $(3, 2, -5/6)$  acquire masses  $\lesssim M_G^2/\Lambda$  [13, 14, 28]. These additional light fields change the RGEs and make gauge unification at  $M_G \gtrsim 10^{15.5} \text{GeV}$  impossible at the 1-loop level if  $\Lambda$  is the Planck scale.

of the relevant Yukawa couplings at  $\mu > M_N$  are [22, 23],

$$16\pi^2 \frac{dy_t}{dt} = y_t \left( \frac{3}{2}y_t^2 - \frac{3}{2}y_b^2 + T - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 \right), \quad (3)$$

$$16\pi^2 \frac{dy_b}{dt} = y_b \left( \frac{3}{2}y_b^2 - \frac{3}{2}y_t^2 + T - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 \right), \quad (4)$$

$$16\pi^2 \frac{dy_\tau}{dt} = y_\tau \left( \frac{3}{2}y_\tau^2 - \frac{3}{2}N_g y_\nu^2 + T - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 \right), \quad (5)$$

$$16\pi^2 \frac{dy_\nu}{dt} = y_\nu \left( \frac{3}{2}N_g y_\nu^2 - \frac{3}{2}y_\tau^2 + T - \frac{9}{20}g_1^2 - \frac{9}{4}g_2^2 \right), \quad (6)$$

$$T \equiv 3y_t^2 + 3y_b^2 + y_\tau^2 + N_g y_\nu^2, \quad (7)$$

where we define  $t \equiv \ln(\mu/M_Z)$ . We also use  $t_N \equiv \ln(M_N/M_Z)$  below. We have neglected Yukawa couplings of the first and second generations of quarks and charged leptons and the light  $T_F$  since they are small ( $y_{T_F} \lesssim \sqrt{m_{T_F} m_\nu}/v \sim 10^{-5}$ ).  $y_\nu$  contributes to  $y_\tau$  negatively, so we can expect that large  $y_\nu$  unifies  $y_b$  and  $y_\tau$ . For initial conditions, we use  $y_b(0)v = 2.86_{-0.06}^{+0.16} \text{ GeV}$ ,  $y_\tau(0)v = 1.74617 \text{ GeV}$ ,  $y_t(0)v = 172.1 \pm 1.2 \text{ GeV}$  [29], where  $v \equiv 2^{-3/4} G_F^{-1/2}$ ,  $G_F = 1.1663787 \text{ GeV}^{-2}$  [25].

We derive an approximate formula for the initial condition  $y_\nu(t_N)$  that satisfies the  $b - \tau$  unification condition,

$$y_b(t_G) = y_\tau(t_G). \quad (8)$$

From Eqs. (4)-(6), we obtain

$$16\pi^2 \frac{d}{dt} \ln \frac{y_\nu y_b^2}{y_\tau^2} = \frac{11}{2} N_g y_\nu^2 + \frac{71}{20} g_1^2 - \frac{9}{4} g_2^2 - 16g_3^2. \quad (9)$$

We neglected  $y_b$  and  $y_\tau$  on the right hand side of Eq. (9), since they are much smaller than other couplings. Here we consider the case  $M_N > m_{OF}$ . In this region,  $y_\nu^2 \gg g_1^2, g_2^2, y_t^2$ , so Eq. (6) can be approximately solved,

$$y_\nu(t) = \left( y_\nu(t_N)^{-2} - \frac{5N_g}{16\pi^2} (t - t_N) \right)^{-1/2}. \quad (10)$$

We found that the difference between this equation and the numerical solution is  $\lesssim 1\%$ . By using this equation and integrating Eq. (9), the GUT relation (8) gives the initial condition of  $y_\nu$ :

$$y_\nu(t_N) = \left[ \frac{16\pi^2}{5N_g(t_G - t_N)} \left( 1 - \left( \frac{y_b(t_N)}{y_\tau(t_N)} \right)^{\frac{10}{3}} \left( \frac{\alpha_1(t_N)}{\alpha_1(t_G)} \right)^{-\frac{71}{24b_1}} \left( \frac{\alpha_2(t_N)}{\alpha_2(t_G)} \right)^{\frac{15}{8b_2}} \left( \frac{\alpha_3(t_N)}{\alpha_3(t_G)} \right)^{\frac{40}{3b_3}} \right) \right]^{1/2}. \quad (11)$$

This is the key formula in this work.  $y_\nu(t_N)$  is plotted in Fig. 2. By solving RGEs (3) to (6) and using the initial condition (11), we found that  $y_b$  and  $y_\tau$  are unified within the experimental errors. Examples are shown in Figs. 3-5.

$y_\nu(t)$  increases monotonically, so we take a perturbativity bound to  $y_\nu(t_G) < \sqrt{4\pi}$  ( $\sim 3.54$ ). It gives an upper bound on the initial condition,

$$y_\nu(t_N) < \left( \frac{1}{4\pi} + \frac{5N_g}{16\pi^2} (t_G - t_N) \right)^{-1/2}. \quad (12)$$

This upper bound determines the maximal values of  $M_N$  (see Fig. 2). Those values and  $y_\nu(t_N)$  are listed in Table 1.

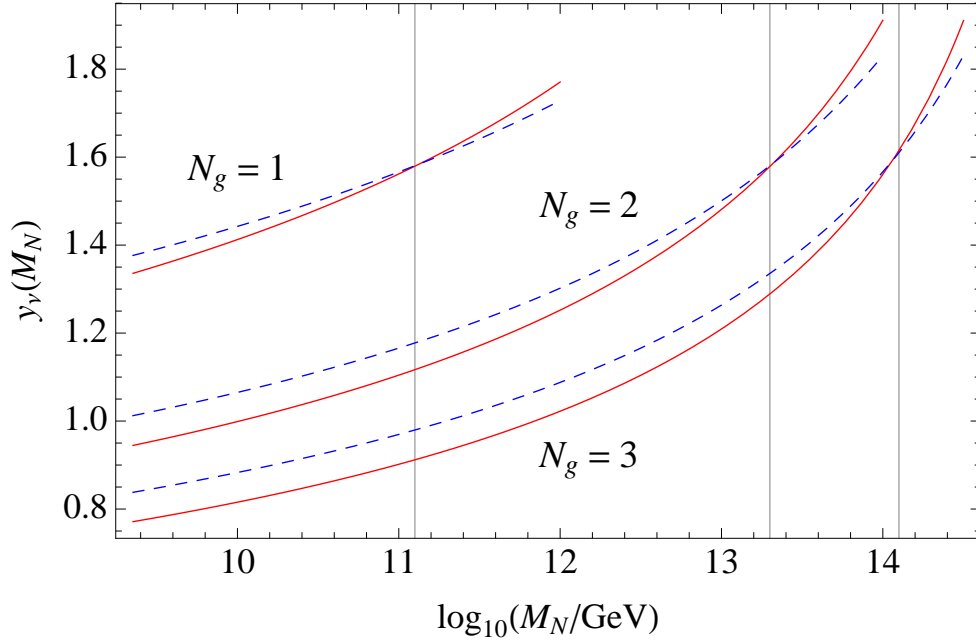


Figure 2: Solid curves show the initial conditions of neutrino Yukawa couplings at  $\mu = M_N$  that realize  $b - \tau$  unification. Dashed curves show upper bounds from perturbativity ( $M_G = 10^{15.5}\text{GeV}$ ). Vertical lines indicate crossing points.

Right-handed neutrinos induce light neutrino mass matrix. The  $(3, 3)$  component of the matrix can be expressed as

$$m_{\nu 33} = -\frac{y_\nu(t_N)^2 v^2}{M_N} \sum_{I=1}^{N_g} e^{2i\theta_I} + m'_{\nu 33}, \quad (13)$$

where we have defined  $Y_{\nu I3} \equiv y_\nu e^{i\theta_I}$  and  $m'_\nu$  is possibly an induced mass by other fields such as  $T_F$ . The most natural case seems to be the  $N_g = 3$  case because  $M_N$  can be close to the seesaw scale (see Table 1). Even if  $M_N \ll 10^{15}\text{GeV}$ , we can obtain  $|m_{\nu 33}| = O(0.01-0.1)\text{eV}$  by tuning  $\theta_I$  or  $m'_\nu$ .

We do not check the stability of the electroweak vacuum here. If  $y_\nu$  is  $O(1)$ , it may make Higgs quartic coupling  $\lambda$  negative for  $t > t_N$ . On the other hand, there are many scalar fields in the  $SU(5)$  model because we need adjoint scalar  $24_H$  to break  $SU(5)$  down to  $G_{\text{SM}}$ . Those scalar fields contribute positively to the running of  $\lambda$  [30–32], so the stability depends on their couplings.  $T_F$  and  $T_H$  make  $\alpha_2$  larger and also contribute to  $\lambda$  in the positive direction [33] (this effect may be small). The vacuum stability is not trivial in our model.

*Summary.* We have studied the effect of the neutrino Yukawa couplings  $y_\nu$  on the running of  $y_b$  and  $y_\tau$ . We have found the  $y_\nu(t_N)$  and  $M_N$  that can satisfy the GUT relation  $y_b(t_G) = y_\tau(t_G)$ . If three right-handed neutrinos exist,  $b - \tau$  unification is possible with their masses close to the seesaw scale.

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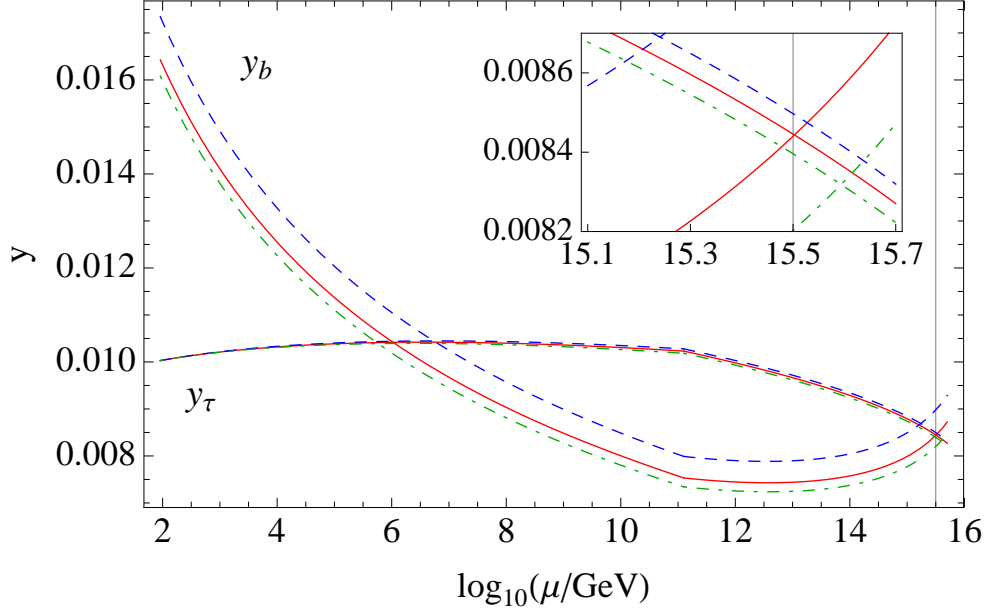


Figure 3: Running of  $b$  and  $\tau$  Yukawa couplings in the case  $N_g = 1$ ,  $M_N = 10^{11.1}\text{GeV}$ ,  $M_G = 10^{15.5}\text{GeV}$ .  $y_\nu(t_N) = 1.58$  is calculated by Eq. (11). Dashed and dot-dashed lines show errors given in Ref. [29]. Vertical lines are drawn at  $\mu = M_G$ .

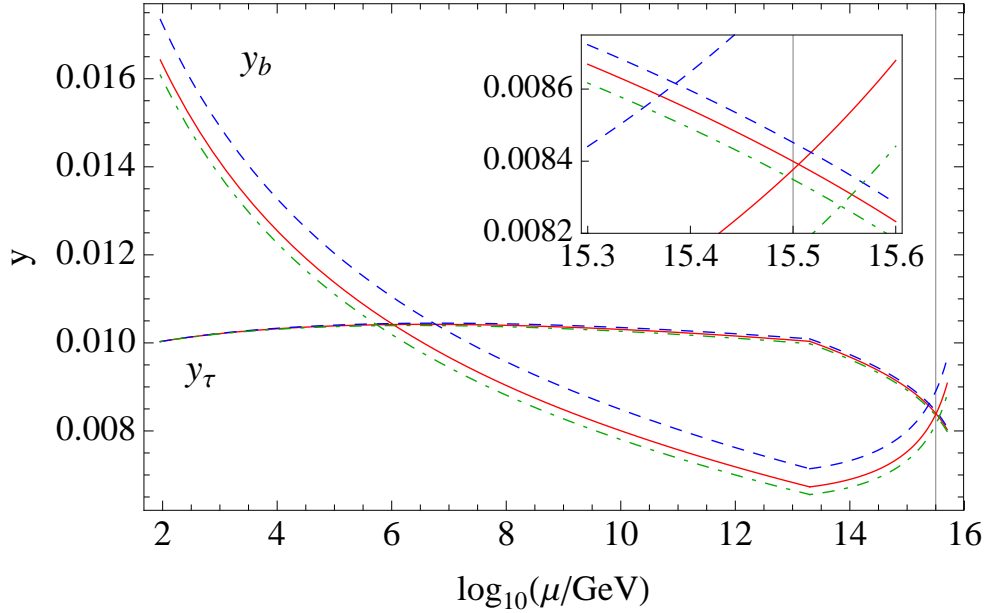


Figure 4: Running of  $b$  and  $\tau$  Yukawa couplings in the case  $N_g = 2$ ,  $M_N = 10^{13.3}\text{GeV}$ . Other conditions are same as in Fig. 3.

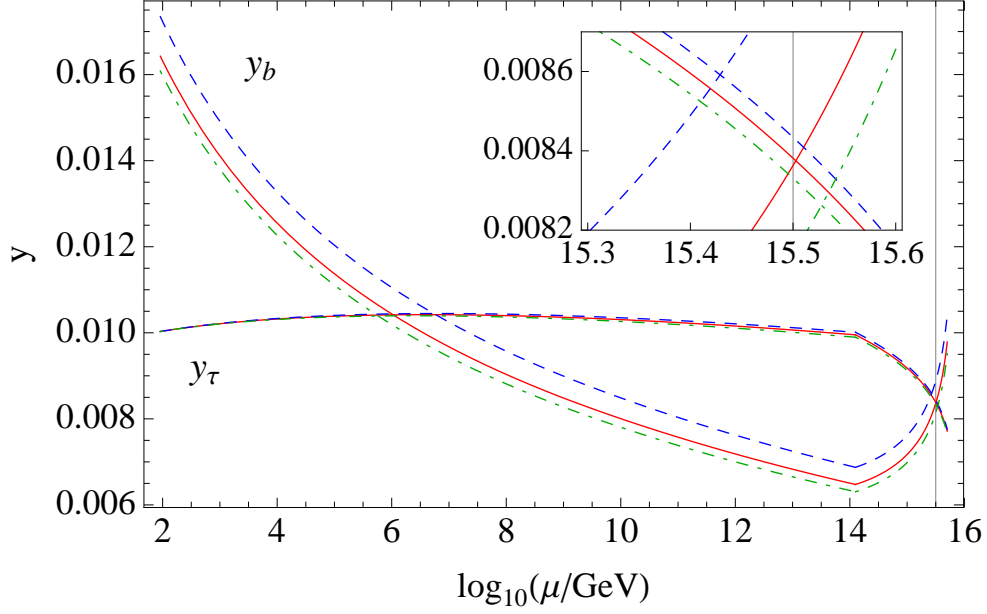


Figure 5: Running of  $b$  and  $\tau$  Yukawa couplings in the case  $N_g = 3$ ,  $M_N = 10^{14.1}\text{GeV}$ . Other conditions are same as in Fig. 3.

Table 1: Maximal masses of the right-handed neutrinos and neutrino Yukawa couplings that realize  $b - \tau$  unification. “Central” is the case of all experimental parameters taken to the best-fit values. “Best” indicates the case of maximal  $y_b(M_Z)$ , minimal  $y_t(M_Z)$  and  $\alpha_3(M_Z)$ . “Worst” is the opposite case to “Best”.

$N_g$	$\log_{10}(M_N/\text{GeV})$			$y_\nu(t_N)$		
	Worst	Central	Best	Worst	Central	Best
$(M_G = 10^{15.5}\text{GeV})$						
1	10.7	11.1	12.1	1.53	1.58	1.75
2	13.1	13.3	13.8	1.53	1.58	1.75
3	13.9	14.1	14.4	1.53	1.62	1.77
$(M_G = 10^{16.0}\text{GeV})$						
1	10.4	10.9	12.0	1.43	1.49	1.64
2	13.2	13.5	14.0	1.43	1.50	1.64
3	14.1	14.3	14.7	1.42	1.49	1.66

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